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Reconciling sterile neutrinos with big bang nucleosynthesis

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Abstract

We re-examine the big bang nucleosynthesis (BBN) bounds on the mixing of neutrinos with sterile species. These bounds depend on the assumption that the relic neutrino asymmetry L_ν is very small. We show that for L_ν large enough (greater than about 10^{-5}) the standard BBN bounds do not apply. We apply this result to the sterile neutrino solution to the atmospheric neutrino anomaly and show that for $L_\nu > 7 \times 10^{-5}$ it is consistent with BBN. The BBN bounds on sterile neutrinos mixing with electron neutrinos can also be weakened considerably.

The solar neutrino deficit [1], atmospheric neutrino anomaly [2], and LSND experiment[3] can all be viewed as evidence for non-zero neutrino masses and oscillations. It does not seem possible to explain all these anomalies with the three known neutrino species and thus new neutrino species might exist. Given that new ordinary weakly interacting neutrino species are ruled out by LEP, sterile neutrinos (ν_s) are a natural candidate. There are essentially two types of sterile neutrinos that can be envisaged. Firstly, there are sterile states which either have no gauge interactions, or interactions which are much weaker than the usual weak interactions [4]. Alternatively, it is possible to envisage neutrinos which do not have significant interactions with ordinary matter but do have significant interactions with themselves. An interesting example of the latter is given by mirror neutrinos which interact with themselves only through mirror weak interactions which have the same strength as ordinary weak interactions [5].

However, for both sterile and mirror neutrinos there are apparently quite stringent bounds if they are required to be consistent with standard big bang cosmology. Assuming that the number of effective neutrino species present during nucleosynthesis is bounded to be less than 4, then the mixing angle (θ_0) and the squared mass difference (δm^2) for a sterile neutrino mixing with one of the known neutrinos is bounded by [6]:

$$\begin{aligned} \delta m^2 \sin^2 2\theta_0 &\lesssim 5 \times 10^{-6}\ eV^2,\ \nu = \nu_e, \\ \delta m^2 \sin^2 2\theta_0 &\lesssim 3 \times 10^{-6}\ eV^2,\ \nu = \nu_{\mu,\tau}. \end{aligned} \tag{1}$$

These bounds arise by demanding that oscillations do not bring the sterile neutrino into equilibrium with the known neutrinos. Electron neutrinos must also not be depleted too much by oscillations after decoupling, during the BBN epoch, because then the freeze out temperature for neutron-proton transitions is increased. For maximal mixing, the bound on δm^2 is extended to $\delta m^2 \lesssim 10^{-8}\ eV^2$ [6]. These "bounds" would appear to exclude the region of parameter space required to explain the atmospheric neutrino anomaly in terms of $\nu_\mu - \nu_s$ oscillation ($\delta m^2 \simeq 10^{-2} eV^2$, $\sin^2 2\theta_0 \simeq 1$), and would restrict the parameter space required to explain the solar neutrino deficit in terms of $\nu_e - \nu_s$ oscillation. An important assumption in deriving the bounds Eq.(1) is that the relic neutrino asymmetries could be neglected. However, the neutrino asymmetries cannot be measured, and at present the origin of particle asymmetries is not fully understood. Only the extremely weak bound

$L_{\nu_\alpha} \lesssim 10^3$ can be derived by demanding that the neutrinos do not violate the upper limit on the total energy density of the universe. The purpose of this letter is to re-examine the BBN bounds on ordinary-sterile neutrino mass and mixing for arbitary neutrino asymmetries. In particular, we will show that for neutrino asymmetries larger than about 7×10^{-5} the standard big bang model is consistent with sterile neutrinos mixing with muon neutrinos with parameters suggested by the atmospheric neutrino anomaly.

Let us first examine the ordinary neutrino (ν_α, $\alpha = e, \mu, \tau$) oscillating with a sterile neutrino (ν_s) in vacuum. Oscillations can occur if the weak eigenstate neutrino and sterile neutrino are each linear combinations $\nu_\alpha = \cos\theta_0 \nu_1 + \sin\theta_0 \nu_2$ and $\nu_s = -\sin\theta_0 \nu_1 + \cos\theta_0 \nu_2$ of mass eigenstates $\nu_{1,2}$. An ordinary neutrino of momentum p will then oscillate in vacuum after a time t with probability

$$|\langle \nu_\alpha(t)|\nu_s\rangle|^2 = \sin^2 2\theta_0 \sin^2\left(\frac{t}{L_{osc}}\right), \tag{2}$$

where [7]

$$L_{osc} = \frac{2p}{\delta m^2} \equiv \frac{1}{\Delta_0}. \tag{3}$$

However, in the early universe oscillations occur in a plasma. For $\nu_\alpha - \nu_s$ ($\alpha = e, \mu, \tau$) oscillations in a plasma of temperature T, the matter and vacuum oscillation parameters are related by [8]

$$\sin^2 2\theta_m = \frac{\sin^2 2\theta_0}{1 - 2z\cos 2\theta_0 + z^2},$$

$$\Delta_m^2 = \Delta_0^2(1 - 2z\cos 2\theta_0 + z^2), \tag{4}$$

where $z = 2\langle p\rangle\langle V_\alpha - V_s\rangle/\delta m^2$ and $\langle V_{\alpha,s}\rangle$ are the effective potentials due to the interactions of the neutrinos with matter ($\langle p\rangle \simeq 3.15T$). For a truly sterile neutrino $V_s = 0$. (For neutrinos which have only self interactions, e.g. mirror neutrinos [5], V_s can be non-zero. We will comment on this case later.) For a weak eigenstate neutrino ($\nu_\alpha, \alpha = e, \mu, \tau$), V_α is given by [9, 10, 11]

$$V_\alpha = \sqrt{2} G_F N_\gamma \left(L^{(\alpha)} - \frac{A_\alpha T^2}{M_W^2}\right), \tag{5}$$

where G_F is the Fermi coupling constant, M_W is the W boson mass, A_α is a numerical factor given by $A_e \simeq 55$ and $A_{\mu,\tau} \simeq 15.3$ [9, 10]. The neutrino asymmetry $L^{(\alpha)}$ is given by

$$L^{(\alpha)} = L_\alpha + L_{\nu_e} + L_{\nu_\mu} + L_{\nu_\tau}, \tag{6}$$

where $L_\alpha = (N_\alpha - N_{\bar{\alpha}})/N_\gamma$. Since we will be interested in the case where the asymmetries are large (of order 10^{-5} or more) we have neglected the asymmetries in the electrons and protons/neutrons since these are known to be small ($\sim 10^{-10}$).

For $L^{(\alpha)}$ non negligible, the bounds of Eq.(1) can be weakened considerably. This is because the oscillation probability depends on $L^{(\alpha)}$ through the dependence of $\sin^2 2\theta_m$ on $L^{(\alpha)}$ in Eqs.(4,5). The condition that the sterile neutrinos not come into equilibrium is that the interaction rate for sterile neutrinos is less than the expansion rate, i.e. $\Gamma_{\nu_s} < H$. We will assume that there are essentially no sterile neutrinos initially. The rate of production of sterile neutrinos is given by the interaction rate of ordinary neutrinos multiplied by the probability that the neutrino collapses to the sterile eigenfunction, i.e.

$$\Gamma_{\nu_s}(t) = \langle P_{\nu_\alpha} \to \nu_s \rangle_{coll} \Gamma_{\nu_\alpha}, \tag{7}$$

where $\Gamma_{\nu_\alpha} = y_\alpha G_F^2 T^5$, ($y_e \simeq 4.0$ and $y_{\mu,\tau} \simeq 2.9$) [6] and $\langle P_{\nu_\alpha} \to \nu_s \rangle_{coll}$ is given by

$$\langle P_{\nu_\alpha} \to \nu_s \rangle_{coll} = \sin^2 2\theta_m \langle \sin^2 \frac{x}{L_{osc}^{(m)}} \rangle, \tag{8}$$

where x is the distance between collisions. Note that $\langle x \rangle \equiv L_{int} = 1/\Gamma_{\nu_\alpha}$ where L_{int} is the mean distance between interactions. From Eq.(4) it is easy to see that the production rate of sterile neutrinos is significantly suppressed (because $\sin^2 2\theta_m \ll \sin^2 2\theta_0$) for temperatures above about 12 MeV (given $\delta m^2 \sim 10^{-2}\ eV^2$), independently of the magnitude of $L^{(\alpha)}$ (except in a resonance region where $\sin^2 2\theta_m = 1$, as we shall discuss later). Below about 12 MeV, $\sin^2 2\theta_m$ approaches its vacuum value (unless $L^{(\alpha)}$ is non negligible). In the standard scenario[6], it is in this region where oscillations can occur and potentially bring the sterile neutrino into equilibrium.

However, for $L^{(\alpha)}$ non negligible, the production rate of sterile neutrinos can continue to be suppressed in the region below 12 MeV. Assuming $|L^{(\alpha)}| > 10^{-5}$ (which we will discuss later), the condition $L_{int}/L_{osc}^{(m)} \gg 1$ always holds off resonance (for temperatures above the decoupling temperature). This means that $\langle \sin^2(L_{int}/L_{osc}^{(m)}) \rangle$ averages to $1/2$. Using this result, and Eqs.(7,8) [with $\sin^2 2\theta_m$ given by Eq.(4)], we can now calculate the production rate of sterile neutrinos for the general case with $L^{(\alpha)}$ non-zero. Demanding that this interaction rate be less than the expansion rate, i.e. $\Gamma_{\nu_s} \lesssim H \simeq 5.5T^2/M_P$, we find for large $L^{(\alpha)}$ (where the second term in Eq.(5) can be neglected) and maximal mixing

$$(\delta m^2)^2 \lesssim 79 G_F^2 T^2 N_\gamma^2 \left[L^{(\alpha)}\right]^2 \left(\frac{y_\alpha M_P G_F^2 T^3}{11} - 1\right)^{-1}, \tag{9}$$

where $M_P \simeq 1.2 \times 10^{22}\ MeV$ is the Planck mass. Note that in the case where the mixing is not maximal, the bound is even more stringent. Clearly, oscillations from ordinary to sterile neutrinos which occur after the kinetic decoupling temperature (T_{dec}) do not significantly affect the energy density of the universe. For temperatures above T_{dec}, the most stringent bound occurs at the decoupling temperature. For ν_e, $T_{dec} \simeq 2.6$ MeV and for $\nu_{\mu,\tau}$, $T_{dec} \simeq 4.4$ MeV, which leads to the bounds:

$$\begin{aligned} |\delta m^2| &< 4 \times 10^2 |L^{(e)}| eV^2,\ \nu = \nu_e \\ |\delta m^2| &< 1.6 \times 10^3 |L^{(\mu,\tau)}| eV^2,\ \nu = \nu_{\mu,\tau}. \end{aligned} \tag{10}$$

These bounds replace those of Eq.(1) in the case where $|L^{(\alpha)}|$ is large [$|L^{(\alpha)}| > 10^{-5}$].

In the case of electron neutrino oscillations into sterile neutrinos a more stringent bound comes by requiring that the electron neutrinos are not depleted significantly down to temperatures where the protons and neutrons go out of equilibrium (which is about 0.8 MeV in the standard scenario). For maximal mixing, the bound is $\delta m^2 < 10^{-8}\ eV^2$ if $L^{(e)}$ is negligible[6]. However in the case where $|L^{(e)}|$ is not negligible ($> 10^{-5}$), the situation is changed. If we demand that $\sin^2 2\theta_m \lesssim 1/10$ for $T \geq 0.8$ MeV (which means that the $\nu_e - \nu_s$ oscillations are severely suppressed for $T \geq 0.8$ MeV) we find in the case of maximal mixing that

$$|\delta m^2| \lesssim 2\sqrt{2} |L^{(e)}| G_F N_\gamma T \lesssim 3.3 |L^{(e)}|\ eV^2. \tag{11}$$

This bound for $\nu_e - \nu_s$ oscillations is clearly more stringent than Eq.(10).

In the above analysis we have assumed that the value of $L^{(\alpha)}$ is fixed. However in reality $L^{(\alpha)}$ is in general not constant. Oscillations can change its value. There are only two regions where oscillations are important. First, there is the region near the high temperature resonance (in this region $\sin^2 2\theta_m = 1$). The other region where oscillations can significantly change $L^{(\alpha)}$ is at the low temperature region $T \approx T_{dec}$. Oscillations can change $L^{(\alpha)}$ in this region because the production rate of sterile neutrinos is not so strongly suppressed (recall $\sin^2 2\theta_m \to \sin^2 2\theta_0$ as $T \to 0$). We now consider each of these regions in turn.

We will assume for definiteness that $L^{(\alpha)}$ is positive (unless explicitly stated otherwise). The production rate of sterile neutrinos is given by Eqs.(7,8), with $\sin^2 2\theta_m$ given by Eqs.(4,5). It is important to observe that for $L^{(\alpha)} > 0$ the sterile neutrino production rate (Γ_{ν_s}) is always greater than the sterile antineutrino production rate ($\Gamma_{\bar{\nu}_s}$). This behaviour is due to the relative minus sign in Eq.(5). Thus, the effect of the oscillations will always reduce the size of $L^{(\alpha)}$. The condition for $\nu_\alpha - \nu_s$ oscillation resonance ($\theta_m = \pi/4$) is that

$$(V_\alpha - V_s) = \Delta_0 \cos 2\theta_0, \tag{12}$$

implying that the resonance temperature is[11]

$$T_{res}^2 = \frac{L^{(\alpha)} M_W^2}{A_\alpha} - \frac{\Delta_0 \cos 2\theta_0 M_W^2}{A_\alpha \sqrt{2} G_F N_\gamma}. \tag{13}$$

Observe that strictly, the right-hand side of Eq.(13) is a function of temperature so that we must solve Eq.(13) for the resonance temperature. However, it turns out we will only be interested in the high temperature resonance and quite large values of $L^{(\alpha)}$, i.e. $L^{(\alpha)} \gtrsim 10^{-5}$, and in this case the second term on the right-hand side of Eq.(13) can be neglected (for $\delta m^2 \leq 1\ eV^2$).

Observe that for $L^{(\alpha)} > 0,\ \delta m^2 > 0$, there is no resonance for antineutrinos while for $L^{(\alpha)} > 0,\ \delta m^2 < 0$ the resonance for antineutrinos occurs at very low temperatures. [Note that for $\delta m^2 \leq 10^{-2} eV^2$, $L^{(\alpha)} \geq 10^{-5}$ this low temperature resonance occurs at temperatures below the kinetic decoupling temperature and thus it can be neglected in our analysis.] Of more importance is the "high temperature" resonance which, for $L^{(\alpha)} > 0$ only occurs for neutrinos (for $L^{(\alpha)} < 0$ the high temperature resonance only occurs for antineutrinos). The effect of the high temperature resonance is rather interesting. For an initial $L^{(\alpha)}$ less than a certain "critical" value (to be determined later) $L^{(\alpha)}$ will evolve to zero. This is essentially because the oscillations near the resonance are so numerous as to continually lower the resonance temperature so that the system cannot actually pass through the resonance.

However, for $L^{(\alpha)}$ large enough there will be a critical point where the expansion of the universe is more important than the change in $L^{(\alpha)}$ due to oscillations. This behaviour has been studied numerically in Ref.[10, 12]. Below we show how this behaviour can be understood and we derive an analytic approximation for the critical value of $L^{(\alpha)}$.

If the change in the resonance temperature due to oscillations is greater than the width of the resonance, then the resonance will be moved to lower and lower temperatures, until the lepton number is reduced to near zero. However if the change in the resonance temperature due to oscillations during the resonance is less than the width of the resonance [13], then this will ensure the system passes briefly through the resonance. Having passed through the resonance, the value of $L^{(\alpha)}$ will not change significantly until much lower temperatures, as discussed earlier. The condition that the system passes through the resonance is that

$$\delta T_{res} \lesssim \Delta T, \tag{14}$$

where δT_{res} is the change in the resonance temperature due to the oscillations as the system passes through the resonance, while ΔT is the width of the resonance. The resonance temperature T_{res} is related to the lepton number asymmetry through Eq.(13) so that

$$\delta T_{res} = \frac{M_W \delta L^{(\alpha)}}{2\sqrt{A_\alpha L_{(\alpha)}}}. \tag{15}$$

Now, $\delta L^{(\alpha)}$ is simply the reaction rate Γ_{ν_s} for ordinary neutrinos converting into sterile neutrinos multiplied by the time it takes for the system to pass through the resonance Δt, i.e. $\delta L^{(\alpha)} = -\Gamma_{\nu_s}\Delta t$. Note that the resonance width measures defined in terms of the temperature and the time, ΔT and Δt, are related to each other using the time-temperature relation of the early universe: $t \simeq M_P/11T^2$ implies $\Delta t \simeq -M_P \Delta T/5.5T^3$. Hence, the condition that the system will pass through the resonance is that

$$\frac{M_W M_P \Gamma_{\nu_s}}{11T^3\sqrt{A_\alpha L^{(\alpha)}}} \lesssim 1. \tag{16}$$

Note that for $L^{(\alpha)} > 10^{-5}$, $T_{res} > 35$ MeV. It is easy to verify that at the resonance, $L_{int}/L_{osc}^{(m)} \ll 1$ and hence $\sin^2(L_{int}/L_{osc}^{(m)})\rangle \simeq L_{int}^2/L_{osc}^{(m)2}$. Using this result, and Eqs. (7,8), we can calculate Γ_{ν_s} at the resonance:

$$\Gamma_{\nu_s}|_{res} = \frac{\Delta_0^2}{y_\alpha G_F^2 T^5}. \tag{17}$$

Substituting $\Gamma_{\nu_s}|_{res}$ into Eq.(16), we obtain

$$L_{crit}^{(\alpha)} \simeq \left(\frac{(\delta m^2)^4 M_P^2 A_\alpha^9}{2 \times 10^5 y_\alpha^2 G_F^4 M_W^{18}}\right)^{\frac{1}{11}}. \tag{18}$$

Note that $L_{crit}^{(\alpha)}$ is independent of the vacuum mixing angle θ_0. For $L^{(\alpha)} < L_{crit}^{(\alpha)}$ the change in the resonance temperature is greater than the width of the resonance. This means that the resonance dynamically evolves to later and later times, with $L^{(\alpha)}$ moving closer and closer to zero. For $L^{(\alpha)} > L_{crit}^{(\alpha)}$ the change in the resonance temperature is less than the width of the resonance, so the system passes through the resonance. Having passed through the resonance the value of $L^{(\alpha)}$ remains approximately unchanged (until much later times as will be discussed later). Thus requiring $L^{(\alpha)} > L_{crit}^{(\alpha)}$ we find

$$\begin{aligned} L^{(e)} &> 9.6 \times 10^{-6} \text{ for } \delta m^2 \leq 10^{-4}\ eV^2, \\ L^{(\mu,\tau)} &> 1.9 \times 10^{-5} \text{ for } \delta m^2 \leq 10^{-2}\ eV^2. \end{aligned} \tag{19}$$

It is useful to compare our analytic expression Eq.(18) with the numerical work of Ref.[12]. They find numerically that for $\delta m^2 = 10^{-4}\ eV^2$ an initial asymmetry of $L^{(e)} = 10^{-5}$ remains unchanged on passing through the resonance. They also obtain that for $\delta m^2 = 10^{-3}\ eV^2$ an initial asymmetry $L_0^{(e)} = 10^{-5}$ leads $L^{(e)}$ to evolve to zero. These results are consistent with our analytic expression Eq.(18). [Putting in $\delta m^2 = 10^{-4}\ eV^2$ in Eq.(18), we find $L_{crit}^{(e)} \simeq 9.6 \times 10^{-6} < L_0^{(e)}$ while for $\delta m^2 = 10^{-3}\ eV^2$ we find $L_{crit}^{(e)} \simeq 2.2 \times 10^{-5} > L_0^{(e)}$.]

As a consistency test, we should check that the change in $L^{(\alpha)}$ on passing through the resonance is small compared with the initial value of $L^{(\alpha)}$. To work out the change in $L^{(\alpha)}$ on passing through the resonance, we must work out the width of the resonance. Note that the width of the resonance is larger than might be expected. Actually, the interaction rate at the centre of the resonance $\theta_m = \pi/4$ is equal to the interaction rate anywhere in the region where $L_{osc}^{(m)} = \Delta_m^{-1} \gg L_{int}$. To see this observe that the interaction rate is given by,

$$\Gamma_{\nu_s} = \Gamma_{\nu_\alpha} \sin^2 2\theta_m \sin^2 \left(\frac{L_{int}}{L_{osc}}\right) = y_\alpha G_F^2 T^5 \frac{\Delta_0^2}{\Delta_m^2} \sin^2 \left(\frac{\Delta_m}{y_\alpha G_F^2 T^5}\right). \qquad (20)$$

Thus, $\Gamma_{\nu_s} = \Gamma_{\nu_s}|_{res}$ [defined in Eq.(17)] provided $\Delta_m \ll L_{int}^{-1} = y_\alpha G_F^2 T^5$. Using Eqs.(13-15), it is easy to show that the change in $L^{(\alpha)}$ on passing through the resonance is related to the resonance width by $\delta L^{(\alpha)}/L^{(\alpha)} = 2\Delta T/T$. Calculating ΔT using $\Delta_m(T_{res} + \Delta T/2) \simeq y_\alpha G_F^2 T_{res}^5$, we find that $\delta L^{(\alpha)}/L^{(\alpha)} \leq 0.04$ (0.10) for $\nu_\alpha = \nu_e$ $(\nu_\alpha = \nu_{\mu,\tau})$. Thus, the the change in $L^{(\alpha)}$ through the resonance is always at least an order of magnitude smaller than $L^{(\alpha)}$.

Note that the calculation of $L_{crit}^{(\alpha)}$ assumes that the sterile neutrino has no significant interactions. In the case of a neutrino which has only significant interactions with itself (such as a mirror neutrino [5]) then V_s is unequal to zero. In this case $L^{(\alpha)}$ does not evolve to zero but evolves so that $V_\alpha - V_s \approx 0$. This does not affect the resulting analysis, since the interaction rate depends on $V_\alpha - V_s$ rather than V_α.

We now consider the low temperature region $T \approx T_{dec}$ in which oscillations can also potentially erase $L^{(\alpha)}$. To calculate $\delta L^{(\alpha)}$ in this region, we must integrate $\delta L^{(\alpha)} = (\Gamma_{\nu_s} - \Gamma_{\overline{\nu}_s})\delta t$, using $\Gamma = (1/2)\Gamma_{\nu_\alpha} \sin^2\theta_0/(1 - 2z\cos 2\theta_0 + z^2)$ where z differs between ν and $\overline{\nu}$ by $L^{(\alpha)} \to -L^{(\alpha)}$. Approximating $1 - 2z\cos 2\theta_0 + z^2$ by z^2, which is approximately valid for $L^{(\alpha)} > 10^{-5}$ and $\delta m^2 \leq 10^{-2}\ eV^2$ $(10^{-5}\ eV^2)$ for $\alpha = \mu, \tau$ and $T \geq 4.4\ MeV$ ($\alpha = e$ and $T \geq 0.8\ MeV$), we find that

$$\left(L_{\rm final}^{(\alpha)}\right)^4 \simeq \left(L_{\rm initial}^{(\alpha)}\right)^4 - \frac{\sin^2\theta_0 A_\alpha y_\alpha M_P (\delta m^2)^2}{5.5 M_W^2 T_f^3} \ > \ (10^{-5})^4, \qquad (21)$$

where the last inequality comes by demanding that $L_{\rm final}^{(\alpha)} > 10^{-5}$ so that the bounds Eqs.(10, 11) remain valid. Note that the strongest tendency towards erasure of $L^{(\alpha)}$ occurs at the low temperature end of the integration region. For $\alpha = \mu, \tau$, $T_f = T_{dec} \simeq 4.4\ MeV$. For $\alpha = e$, we require $T_f \simeq 0.8\ MeV$ because we require that $L^{(e)}$ not be erased above the temperature in which the protons and neutrons are kept in equilibrium. Evaluating Eq.(21) we find that in the most stringent case of maximal mixing:

$$L^{(e)}_{\rm initial} > 4 \times 10^{-5} \text{ for } \delta m^2 \leq 10^{-4} \ eV^2,$$
$$L^{(\mu,\tau)}_{\rm initial} > 7 \times 10^{-5} \text{ for } \delta m^2 \leq 10^{-2} \ eV^2. \tag{22}$$

These bounds are slightly more stringent (for maximal mixing) than those obtained earlier from requiring that $L^{(\alpha)}$ not be erased due to the high temperature resonance.

Thus, we conclude that if $L^{(\alpha)}$ satisfies Eqs.(19) and (22) then $L^{(\alpha)}$ is not erased either at high temperatures or at low temperatures, and hence for $L^{(\alpha)}$ satisfying Eqs.(19) and (22) the bounds Eqs.(10, 11) hold. We conclude that for $L^{(\alpha)}$ satisfying Eqs.(19) and (22) the sterile neutrino solution to the atmospheric neutrino anomaly is consistent with BBN. Also, the large angle MSW $\nu_e - \nu_s$ oscillation solution to the solar neutrino problem is also consistent with BBN as is the maximal mixing vacuum oscillation solution (see e.g. Ref.[5]).

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